On variants of the $\mathbb{C} \oplus M_{2\times 2}(\mathbb{C}) \oplus M_{3\times 3}(\mathbb{C})$ NCG model of elementary particles

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Abstract

We investigate restrictions to be imposed in the NCG $C + M_2 + M_3$ model to make it to fit with phenomenological data. Under strong conditions over the NCG field a leptophobic Z' boson is got.

Recent work [8] shows that naive application of Connes's scheme[7] to the $\mathbb{C} \oplus M_{2\times 2}(\mathbb{C}) \oplus M_{3\times 3}(\mathbb{C})$ algebra drives to a model of elementary particles which has not an easy phenomenological fit, nor a trivial method to remove anomalies.

In this short letter, we point out that results can be best fitted if we take into account the difference between quark and leptonic sectors. In Connes' work [1], the bialgebra $\mathbb{C} \oplus M_{2\times 2}(\mathbb{C}), \mathbb{C} \oplus M_{3\times 3}(\mathbb{C})$ is rejected because we need to get the quark yukawa couplings of the standard model, and such condition is automatically achieved if we take the bialgebra to be $\mathbb{C} \oplus \mathbb{H}, \mathbb{C} \oplus M_{3\times 3}(\mathbb{C})$. But no restriction was really needed for the lepton sector. So our path of search can start from $\mathbb{C} \oplus M_{2\times 2}(\mathbb{C}), \mathbb{C} \oplus M_{3\times 3}(\mathbb{C})$ and look for conditions restricting the action on quarks to be *quaternionik*.

Indeed, this can be the case if we demand the fields to show some kind of "Independence between actions by $\mathbb{C}$ and by $M_{2\times 2}(\mathbb{C})$"

Plan of letter is as follows: First we look some justifications for shrinking the $M_{2\times 2}(\mathbb{C})$ action to be $\sim \mathbb{H}$ over the quarks. We develop the calculation in the bimodule formalism, where it is simpler to separate quark and leptons. Then unimodularity conditions are applied and we examine the resulting fields, then relating it to the phenomenological ones. Finally, we conclude with some comments about where to extend this toy model towards.

Remember that a field A is a first order operator $A = \sum a[D, a']$ which is self-adjoint under the $*$-involution (see [1, 2, 9] for details). For models of type $\mathcal{A} = \mathcal{C}(\mathcal{M}) \otimes \mathcal{A}_{\mathcal{F}}$, i.e, an algebra of continuous functions times a finite matrix algebra, this operator decomposes in a term due to the $\mathcal{A}_{\mathcal{F}}$ and other coming from the one of continuous functions. The $*$ involution acts as adjunction in the finite part and anti-adjuction in the continuous one.

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The finite term of A for the quark part is:

$$\pi_q = \sum \begin{pmatrix} 0 & 0 & m_d^+\lambda(\alpha' - \lambda') & m_d^+\lambda\beta' \\ 0 & 0 & -m_u^+\bar{\lambda}\bar{\beta}' & m_u^+\bar{\lambda}(\bar{\alpha}' - \bar{\lambda}') \\ (\alpha(\lambda' - \alpha') + \beta\bar{\beta}')m_d & (-\alpha\beta' + \beta(\bar{\lambda}' - \alpha'))m_u & 0 & 0 \\ (-\bar{\beta}(\lambda' - \alpha') + \bar{\alpha}\bar{\beta}')m_d & (\bar{\beta}\beta' + \bar{\alpha}(\bar{\lambda}' - \bar{\alpha}'))m_u & 0 & 0 \end{pmatrix} \tag{1}$$

Where $x, \bar{x}$ were conjugate complex numbers in the $\mathbb{C} \oplus \mathbb{H}$ model, but now they are independent(We notate the complex conjugate as x^+). For the lepton part, the operator is:

$$\pi_l = \sum \begin{pmatrix} 0 & m_e^+\lambda(\alpha' - \lambda') & m_e^+\lambda\beta' \\ (\alpha(\lambda' - \alpha') + \beta\bar{\beta}')m_e & 0 & 0 \\ (-\bar{\beta}(\lambda' - \alpha') + \bar{\alpha}\bar{\beta}')m_e & 0 & 0 \end{pmatrix} \tag{2}$$

Now, $A = A^*$ implies two restrictions in both parts; namely:

$$\sum \lambda(\alpha' - \lambda') = \sum (\alpha(\lambda' - \alpha') + \beta\bar{\beta}')^+ \tag{3}$$
$$\sum \lambda\beta' = \sum (-\bar{\beta}(\lambda' - \alpha') + \bar{\alpha}\bar{\beta}')^+ \tag{4}$$

and the quark part has two additional conditions (which do not apply to leptons due to the absence of massive neutrino):

$$\sum \bar{\lambda}(\bar{\alpha}' - \bar{\lambda}') = \sum (\bar{\beta}\beta' + \bar{\alpha}(\bar{\lambda}' - \bar{\alpha}'))^+ \tag{5}$$
$$\sum \bar{\lambda}\bar{\beta}' = \sum (\alpha\beta' - \beta(\bar{\lambda}' - \bar{\alpha}'))^+ \tag{6}$$

Note that if we take the algebra of quaternions, the two last equations are simply conjugates of the two former. To clarify calculation, let's define variables that tell us how much the M_2 elements differ from being quaternions:

$$\mu = \bar{\beta} - \beta^+ \tag{7}$$
$$\nu = \bar{\alpha} - \alpha^+ \tag{8}$$

With this notation, let us subtract (5) and (6) from -the conjugates of- (3),(4) respectively. We get the relation

$$\sum \lambda^+\nu' - \nu^+\lambda' = \sum \mu^+\beta'^+ - (\alpha + \nu^+)\nu'^+ - \nu^+\alpha' - \beta\mu' \tag{9}$$
$$\sum \lambda^+\mu' - \mu\lambda' = \sum \beta^+\nu'^+ - \mu\alpha' - (\alpha^+ + \nu)\mu' - \nu\beta'^+ \tag{10}$$

Now we examine the continuous part, which is the one giving the gauge bosons. The part coming from $\mathbb{C}$ is $\Lambda = \sum \lambda d\lambda'$, while the term asociated to $M_{2\times 2}(\mathbb{C})$ has the form

$$Q = \sum \left(\begin{pmatrix} \alpha & \beta \\ -\beta^+ & \alpha^+ \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -\mu & \nu \end{pmatrix} \right) \left(\begin{pmatrix} d\alpha' & d\beta' \\ -d\beta'^+ & d\alpha'^+ \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -d\mu' & d\nu' \end{pmatrix} \right) \tag{11}$$

Over this, the condition $A = A^*$ asks Q to be anti-selfadjoint which implies the following two conditions:

$$\sum -\beta^+ d\mu'^+ + \mu d\beta' - \nu d\alpha'^+ - (\alpha^+ + \nu) d\nu' = \sum \beta d\mu' - \mu'^+ d\beta'^+ + \nu^+ d\alpha' + (\alpha + \nu^+) d\nu'^+ \tag{12}$$

$$\sum -\beta^+ d\nu'^+ = \sum -\mu d\alpha' - \nu d\beta'^+ - (\alpha^+ + \nu) d\mu' \tag{13}$$

Now we can use (9,10) over the two last equations to obtain [1]

$$\sum d(-\nu\lambda'^+ + \lambda\nu'^+) - d(\nu^+\lambda' - \lambda^+\nu') = \sum F(d\alpha, d\beta, d\mu, d\nu, \alpha', \beta', \mu', \nu') \tag{14}$$

$$\sum d(\lambda^+\mu' - \mu\lambda') = \sum G(d\alpha, d\beta, d\mu, d\nu, \alpha', \beta', \mu', \nu') \tag{15}$$

So we can get additional relations between the $\mathbb{C}$ and $M_{2\times 2}(\mathbb{C})$ algebras, which were tautologies in the $\mathbb{C} \oplus \mathbb{H}$ case. It's unclear for us if such restrictions have real relevance after summation. If they had, as they enter through the non-cuaternionic part of M_2, we would choose $\mu = \nu = \mu' = \nu' = 0$ to avoid them.

Anyway, if we assume directly such restriction $M_{2\times 2}(\mathbb{C}) \to \mathbb{H}$ in the quark sector, the representation of A results in a continuous part:

$$\pi_q(\Lambda, V_0) = \begin{pmatrix} \Lambda & & \\ & \bar{\Lambda} & \\ & & V_0 \end{pmatrix}, \Lambda \in i\mathbb{R}, V_0 \in \mathbb{H} \tag{16}$$

Per contra, as (9,10) do not appear in the lepton side, we choose do not restrict it, and the corresponding term is given by:

$$\pi_l(\Lambda, V_0, B) = \begin{pmatrix} \Lambda & \\ & V_0 + B \end{pmatrix}, V_0 \in \mathbb{H};\ \Lambda, B \in i\mathbb{R} \tag{17}$$

With this, the action of the bimodule for the hilbert space $\mathcal{H} = h_l \oplus (h_q \otimes \mathbb{C}^3)$ can be writen (with $K \in M_3, U, \Lambda, B \in i\mathbb{R}, V_0 \in \mathbb{H}$) as:

$$\pi((\Lambda, V_0, B), (U, K)) = (\pi_l(\Lambda, V_0, B) + U) \oplus (\pi_q(\Lambda, V_0) \otimes K) \tag{18}$$

Now, we apply unimodularity conditions in the old style [1, 2]

$$N_g(\Lambda + U) + 2N_g \mathrm{Tr} K = 0 \tag{19}$$

$$2N_g B + 2N_g U + 2N_g \mathrm{Tr} K = 0 \tag{20}$$

N_g being the number of generations.

[1] Additional restrictions coming from the non-emptyness of the kernel of $\pi(\Omega A)^1$ only imply a decrease of freedom in the RHS of eq. (14,15) and do not change the conclusion

From this, we got the relationships

$$\Lambda = U + 2B \tag{21}$$

and

$$(U+B) + \mathrm{Tr}K = 0 \tag{22}$$

Rewriting

$$A_0 = U + B \tag{23}$$

$$K_0 = K + \frac{1}{3}A_0 \tag{24}$$

we finally get [2]

$$\pi(A_0, V, B, K_0) = \begin{pmatrix} 2A_0 & & & & \\ & \frac{2}{3}A_0 + K_0 - B & & & \\ & & -\frac{4}{3}A_0 + K_0 + B & & \\ & & & A_0 + V_0 & \\ & & & & -\frac{1}{3}A_0 + V_0 + K_0 \end{pmatrix} \tag{25}$$

where A_0 coincides with the $U(1)$ field of standard model, with the correct hyperchargues, V_0 is the $SU(2)$ electroweak field, K_0 is the $SU(3)$ color field and B is a new boson field coupling only to quarks.

The resulting model is not anomaly-free. But we are not going to address anomalies (coming from the mixed $U(1)_{A_0} - U(1)_B$ triangles) here. Simply note that no cancellation mechanism seems available in this small framework.

Note that B is leptophobic, as required by recent studies [4] on new electroweak physics. Moreover, we can suppose that its coupling constant, g_2, is the same that the one of the $SU(2)$ electroweak group, as both fields come from the $U(2)$ field associated to the $M_{2\times 2}(\mathbb{C})$ algebra.

New axial and vector currents associated to this, say, Z' field, are zero in the lepton sector. For quarks, we get

$$t_V = +g_2/4 \quad t_A = -g_2/4 \tag{26}$$

on quarks u,c,t, and same with opposed signs for d,s,b:

Doing the quotient by the Z_0 currents, we get the numbers:

- For leptons

$$l_V = 0 \quad l_A = 0 \tag{27}$$

- For quarks u,c,t:

$$t_V = +\frac{1}{2}\frac{\cos^2\theta_w}{\sin\theta_w(\frac{1}{2} - \frac{4}{3}\sin^2\theta_w)} \approx 4.16 \quad t_A = -cos\theta_W \approx -0.87 \tag{28}$$

[2] with basis ($e^R \quad d^R \quad u^R \quad (e,\nu)^L \quad (d,u)^L$)

- For quarks d,s,b:

$$b_V = -\frac{1}{2}\frac{\cos^2\theta_W}{\sin\theta_W(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W)} \approx 2.32 \quad b_A = cos\theta_W \approx 0.87 \tag{29}$$

which we can compare with the experimental fit [4]

$$\pm\, l_V = -2.25 \pm 6.25\frac{M_{Z'}}{1\text{TeV}} \qquad \pm l_A = -0.2 \pm 0.5\frac{M_{Z'}}{1\text{TeV}} \tag{30}$$

$$\pm b_V = -3.45 \pm 20.72\frac{M_{Z'}}{1\text{TeV}} \qquad \pm b_A = +4.58 \pm 9.84\frac{M_{Z'}}{1\text{TeV}} \tag{31}$$

$$\pm c_V = -6.94 \pm 26.6\frac{M_{Z'}}{1\text{TeV}} \qquad \pm c_A = -7.88 \pm 8.46\frac{M_{Z'}}{1\text{TeV}} \tag{32}$$

got from LEP results. We see that the new interaction could fit with the phenomenology, but present limits on Z' mass [3] suggest a slightly higher or more sophisticated coupling.

To summarize, we draw three conclusions:

- It seems valid, at least operationally, to restrict the representation of the fields in the $\mathbb{C} \oplus M_{2\times 2}(\mathbb{C})$ algebra to be the ones of $\mathbb{C} \oplus \mathbb{H}$ in the quark subspace.
- From a representation of this kind, $\mathbb{C} \oplus \mathbb{H}$ over quarks, $\mathbb{C} \oplus M_{2\times 2}(\mathbb{C})$ on leptons, both the standard and the "bizarre" [10] distribution of hyperchargues appear.
- The new model continues being compatible with the experimental data.

Anomaly conditions have not been examined here. Same with the Higgs, which in this setup takes a delicate shape; we need to look how many higgses we have, and which one has the correct quantum numbers to confer mass to the new field.

Such questions are delicate to establish in the model, but the main goal of this letter is only to point a possibility. In fact, we are doing in some sense a leap of faith when jumping from equations (14-15) to result (16), as we assume that such equations have different implications that the one we can get from (3) and the anti-selfadjointness of the diagonal part of the quaternion.

It rests to do some small comments about possible developments. Lets remark again that this presentation is not a definitive one. Serius model building will be done actually in the mood of [7, 5] to incorporate the Tomita operator. As pointed in [5], the final model would be clearly related to $SU_q(2) \otimes SU_q(2)$, not to the single $SU_q(2)$ as it is said to happen here. And perhaps the last word on anomaly cancellation would be say in the framework of a completely unified theory (in the shape of [6]?), where mechanisms as Green-Swartz cancellation[13, 12], horizontal symmetries [11], etcetera, could be available.

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